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PROPAGATION OF ELASTIC WAVES THROUGH A DISCONTINUITY IN A MEDIUM*

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We investigated the problem of the reflection of a plane monochromatic elastic wave from discontinuities of a medium in the form of a contact dislocation and calculated the coefficient of reflection of a wave from the latter.

Investigations of the propagation of acoustic and electromagnetic waves in layered and inhomogeneous media as well as in media with discontinuities are finding increasing use in the solution of many problems. Thus, for example, knowing the dependence of the kinematic and dynamic parameters of elastic waves on the specific features of the distribution of elastic moduli of the material of the medium considered permits one to improve acoustic and radiowave methods of nondestructive testing of structural materials [1, 2], develop various means of transmitting signals in radioengineering [3], and invent new seismic methods of investigating sections of the earth's crust [4, 5].

When the propagation of elastic waves is considered, the medium is usually modeled either by a continuum with continuously varying properties or by a system of solid blocks interacting along the boundaries of their contact, which are represented by smooth surfaces. Recently, however, situations have arisen where the use of such notions leads to results that do not agree with data of full-scale or laboratory experiments.

A way out of this situation was suggested by V.B.Pisetskii on the basis of analysis of experimental data. In his approach the main physical reflectors of elastic waves are discontinuities that have internal structure (contact and other types of dislocations), rather than structureless boundaries of sharp change in the elastic properties of the medium. Hence, it is necessary to determine the dynamic and kinematic parameters of elastic waves reflected from contact dislocations.

In the present work a solution is proposed for the problem of the reflection and refraction of a plane monochromatic elastic wave from one contact dislocation. It is assumed here that a satisfactory representation of the real discontinuity is elastic interaction of two media through a set of discrete elements - supports - whose number is determined by the character of the acting load.

Consider a medium consisting of two semi-infinite elastic plates characterized by the elasticity moduli λ, μ and the density ρ. The plates are separated by a contact dislocation of thickness l (Fig. 1), which consists of supports of thickness h_1 with a spacing h. In general, the supports are located randomly so that the latter assumption is a rather strong one. However, as shown below, using it leads to suitable results.

Let us introduce a coordinate system as shown in Fig. 1. The process of the propagation of elastic waves in the media considered is described by the displacement vector ω_1 for the upper medium ($y > 0$) and ω_2 for the lower one ($y < -l$). We assume that a generalized plane stress state is realized in the plates [6], and therefore for the vectors ω_i we have the Lamé equations

$$\rho \frac{\partial^2 \omega_1}{\partial t^2} = (\lambda^* + \mu)\,\mathrm{div}\,\mathrm{grad}\,\omega_1 + \mu \Delta \omega_1, \qquad y > 0; \tag{1}$$

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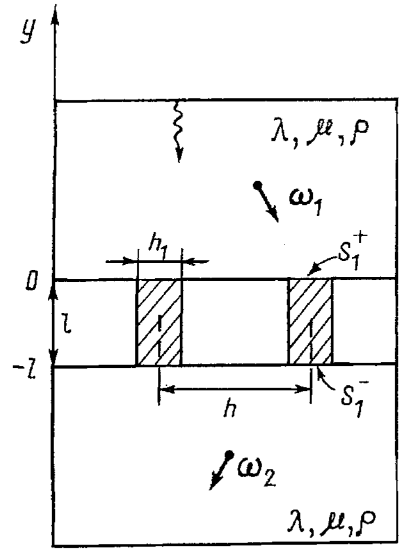

Fig. 1. Schematic representation of the structure of a contact dislocation between two elastic plates.

$$\rho \frac{\partial^2 \omega_2}{\partial t^2} = (\lambda^* + \mu)\,\mathrm{div}\,\mathrm{grad}\,\boldsymbol{\omega}_2 + \mu\Delta\boldsymbol{\omega}_2\,, \quad y < -l\,,\ \lambda^* = 2\lambda\mu/(\lambda + 2\mu)\,. \tag{2}$$

In Eqs. (1) and (2) the plates are considered to be infinite in the directions $x \to \pm\infty$ and $y \to \pm\infty$. Equations (1) and (2) become connected when we take into account the interaction of the elastic continua through the supports. For this purpose, we introduce the simplest model of the behavior of the supports of the contact dislocation when an elastic wave is incident on it.

We assume that when a wave acts on a support, the latter undergoes uniform deformation in the direction of the y axis. If the displacement vector ω_1 at $y = 0$ is $\omega_1 = (0, \omega_1)$ and the displacement vector ω_2 at $y = -l$ is $\omega_2 = (0, \omega_2)$, then the normal stress on the surface of the support $y = 0$ is equal to

$$(\lambda^* + 2\mu)\frac{\partial \omega_1}{\partial y} = (\lambda^* + 2\mu)\frac{\omega_2 - \omega_1}{l}\,, \quad y = 0\,,\ x \in S_i^+\,, \tag{3}$$

where S_i^+ is the upper surface of the i-th support. Correspondingly, at $y = -l$

$$(\lambda^* + 2\mu)\frac{\partial \omega_2}{\partial y} = (\lambda^* + 2\mu)\frac{\omega_2 - \omega_1}{l}\,, \quad y = -l\,,\ x \in S_i^-\,, \tag{4}$$

where S_i^- is the lower surface of the support. Note that when plane waves are incident on a support, we neglect the lateral displacement of the points of the medium and the support at the places of their contact as well as shear stresses.

Normal stresses on the remaining portion of the contact dislocation, as well as shear stresses over the entire boundary of the contact dislocation are equal to zero, namely,

$$\lambda^* \frac{\partial u_1}{\partial x} + (\lambda^* + 2\mu)\frac{\partial \omega_1}{\partial y} = 0\,, \quad y = 0\,,\ x \notin S_i^+\,, \tag{5}$$

$$\lambda^* \frac{\partial u_2}{\partial x} + (\lambda^* + 2\mu)\frac{\partial \omega_2}{\partial y} = 0\,, \quad y = -l\,,\ x \notin S_i^-\,, \tag{6}$$

$$\mu\left(\frac{\partial u_1}{\partial y}+\frac{\partial \omega_1}{\partial x}\right)=0\,,\quad y=0\,,\quad -\infty<x<\infty\,, \tag{7}$$

$$\mu\left(\frac{\partial u_2}{\partial y}+\frac{\partial \omega_2}{\partial x}\right)=0\,,\quad y=-l\,,\quad -\infty<x<\infty\,. \tag{8}$$

Thus, Eqs. (1)-(8) are a full system of relations for determining the characteristics of the waves reflected from a contact dislocation. A solution to (1)-(8) obtained analytically but it has a very awkward form that can hardly be applied to practical calculations. Therefore we will simplify model (1)-(8) in what follows.

Usually in practice the wavelength l greatly exceeds the spacing h between the supports. Therefore, we may introduce averaging over the coordinate x on a scale of length h_2 satisfying the order-of-magnitude relation $h_2 >> h$, $h_2 << l$, namely, the mean value of the quantity a is equal to

$$\bar{a}=\frac{1}{2h_2}\int_{x-h_2}^{x+h_2} a\,dx\,. \tag{9}$$

Let use the notation $\overline{\omega}_i = W_i$, $\overline{u}_i = U_i$. It can be easily understood that the means W_i and U_i are independent of the coordinate x. Moreover, the following relations hold:

$$\frac{\partial W_i}{\partial x}=\overline{\frac{\partial \omega_i}{\partial x}}\,,\quad \frac{\partial W_i}{\partial y}=\overline{\frac{\partial \omega_i}{\partial y}}$$

etc. Therefore, Lamé equations (1), (2) and relations (7), (8) for the shear stresses will take the form

$$\rho\frac{\partial^2 U_1}{\partial t^2}=\mu\frac{\partial^2 U_1}{\partial y^2}\,,\quad y=0\,, \tag{10}$$

$$\rho\frac{\partial^2 U_2}{\partial t^2}=\mu\frac{\partial^2 U_2}{\partial y^2}\,,\quad y<-l\,, \tag{11}$$

$$\rho\frac{\partial^2 W_1}{\partial t^2}=(\lambda^*+2\mu)\frac{\partial^2 W_1}{\partial y^2}\,,\quad y>0\,, \tag{12}$$

$$\rho\frac{\partial^2 W^2}{\partial t^2}=(\lambda^*+2\mu)\frac{\partial^2 W_2}{\partial y^2}\,,\quad y<-l\,, \tag{13}$$

$$\frac{\partial U_1}{\partial y}=0\,,\quad y=0\,;\quad \frac{\partial U_2}{\partial y}=0\,,\quad y=-l\,. \tag{14}$$

Averaging of the left-hand sides of Eqs. (3)-(6) is performed at once and the mean is equal to $(\lambda^*+2\mu)\partial W_i/\partial y$. To calculate the mean of the right-hand sides of Eqs. (3)-(6) we consider the integral

$$I=\frac{\lambda^*+2\mu}{l}\,\frac{1}{2h_2}\int_{x-h_2}^{x+h_2} f(\xi)\,d\xi\,,$$

where

$$f(\xi) = \begin{cases} \omega_2|_{y=-l} - \omega_1|_{y=0}, & \xi \in S_i^+, \\ 0, & \xi \notin S_i^+. \end{cases}$$

The following relation holds:

$$I = \frac{\lambda^* + 2\mu}{l} \frac{1}{2h_2} h_1 \sum_{i=1}^{N_1} (\omega_2 - \omega_1)\Big|_{\xi \in S_i^+}, \tag{15}$$

where N_1 is the number of supports over an interval of length $2h_2$; obviously, $N_1 >> 1$.

It is not difficult to see that relation (15) can be approximately rewritten in the form

$$I \approx \frac{\lambda + 2\mu}{l} h_1 n \frac{1}{2h_2} \int_{x-h_2}^{x+h_2} (\omega_2 - \omega_1)\, d\xi = \frac{\lambda + 2\mu}{l} h_1 n (W_2 - W_1),$$

where $n = N_1/2h_2$ is the number of supports per unit length.

Thus, we obtain two missing boundary conditions for Eqs. (10)-(13):

$$\frac{\partial W_1}{\partial y} = \frac{h_1 n}{l} (W_2|_{y=-l} - W_1), \quad y = 0, \tag{16}$$

$$\frac{\partial W_2}{\partial y} = \frac{h_1 n}{l} (W_2 - W_1|_{y=0}), \quad y = -l. \tag{17}$$

All the characteristics of the contact dislocation are now incorporated in the coefficients of system (10)-(17), and therefore without loss of generality we can transfer boundary conditions (14) and (17) to the boundary $y = 0$.

Suppose a plane normally polarized monochromatic wave $\mathbf{W}_1 = (0, \exp(i\omega t - iky))$ is incident on the contact dislocation ($y = 0$). Obviously, $U_i \equiv 0$,

$$W_1 = \exp(i\omega t - iky) + V \exp(i\omega t + iky), \tag{18}$$

$$W_2 = R \exp(i\omega t - iky), \tag{19}$$

where V is the reflection coefficient. In accordance with Eqs.(12) and (13), we have a relationship between the frequency ω and the wave number k:

$$\omega = \left(\frac{\lambda^* + 2\mu}{\rho}\right)^{1/2} k,$$

after which relations (18) and (19) are the solutions of Eqs. (12) and (13) (Eqs. (10) and (11) hold, since the mean tangential displacements are equal to zero).

Substituting Eqs. (18) and (19) into boundary conditions (16) and (17), we obtain a system of two linear inhomogeneous equations for the amplitudes V and R. The solutions of this system have the form

$$V = \frac{1 + mi}{m^2 + 1}, \quad R = \frac{m(m - i)}{m^2 + 1}, \quad m = \frac{2h_1 n}{lk}. \tag{20}$$

Using the first of Eqs. (20), we determine the modulus r of the reflection coefficient and the phase shift of the reflected wave

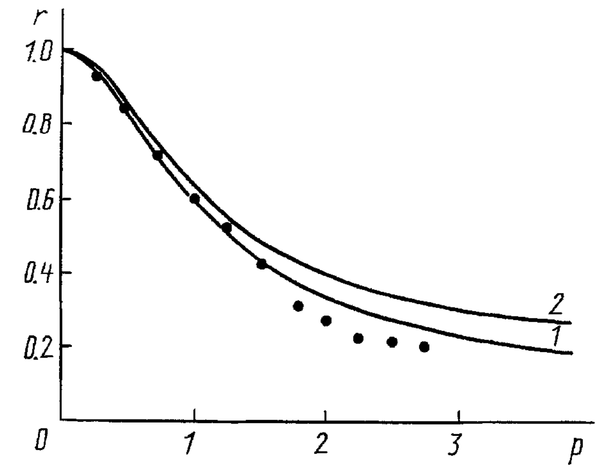

Fig. 2. Dependence of the modulus of the reflection coefficient on the pressure: 1) upper and lower plates with identical elastic properties; 2) plates in which the velocities of propagation of longitudinal vibrations differ twofold; points are experimental data obtained by V.B.Pisetskii.

$$r = |V| = (m^2 + 1)^{-1/2}, \quad \varphi = \arctan m . \tag{21}$$

Relations (21) determine the basic characteristics of the reflected wave from the parameters l, n, h_1 that characterize the contact dislocation.

Let us discuss results obtained by considering two limiting situations. Suppose the number of supports per unit length n is vanishingly small. In this case the upper elastic medium practically does not interact with the lower one and virtually borders on an empty space. Here all the energy of the wave is reflected, and the reflection coefficient is equal to unity. As is seen from Eq. (20), the limit $n \to 0$ corresponds to the approach of the dimensionless number m to zero and of the reflection coefficient r to unity, which corresponds to complete reflection.

Consider the other limiting case where the thickness of the contact dislocation l approaches zero. Now, the both elastic bodies represent a single whole, and the wave will pass without reflection. The limit $l \to 0$ corresponds to an infinite increase in the parameter m and the approach of the refraction coefficient R to unity. The latter means complete passage of the wave. Thus, in both situations the relations derived describe correctly the characteristics of the passage of a plane monochromatic wave.

For applications, e.g., in seismological prospecting, it is of interest to determine the dependence of the coefficient r on the external pressure p (we select the normal lithostatic pressure for the unit of pressure). For this purpose, it is necessary to establish the relationship between the parameter m and the pressure p. As far as we know, this relationship has not been determined theoretically or experimentally. Therefore, here we introduce the dependence between these parameters phenomenologically, proceeding from the following considerations. In the case of complete unloading ($p = 0$), the plates virtually do not interact ($n = 0$), and the parameter m is equal to zero. Under rather heavy loading, the number of contacts n per unit length increases, as does the width of the supports h_1, and the spacing between the plates l decreases. In this case the limit $p \to \infty$ there corresponds to the limit $m \to \infty$. Taking into account the foregoing, we shall adopt the simplest dependence between the indicated quantities m and p: $m = \alpha p$, where the proportionality factor α is to be determined experimentally. The dependence of the reflection coefficient r on the pressure is shown in Fig. 2 (curve 1).

To determine the characteristics of reflected waves we conducted experiments on a plane modeling setup. It incorporates two thin flat organic glass plates separated by a contact dislocation, a loading system, and an acoustic source (Fig. 1). The results of the experiments are shown by the points in Fig. 2 (the parameter α is taken to be equal to 1.4) and agree satisfactorily with the theory suggested in the present work. In conclusion we note that such an approach will make it possible to also obtain the solution of the problem for other models of media with contact dislocations, including models that consider differences between the elastic properties of the plates (Fig. 2), various types of filling of the space between the supports in the dislocation (gas, liquid, viscous medium), and the interaction of a set of dislocations. Investigation of these models and obtaining analytic relationships between characteristics of the reflected waves and parameters of the dislocations allow one to create a fundamentally new basis for solving inverse applied problems of seismic surveying so as to predict the dislocation structure of the earth's crust and parameters of stress states in it from the characteristics of elastic waves.

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